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Managing Uncertainty: Financial, Actuarial and Statistical Modeling

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ABSTRACT

In this paper we give the outline of a research project developed in a cooperation between the actuarial, financial and statistical research groups of the Faculty of Economics and Applied Economics and the research group on statistics in the Mathematical Department. The main purpose consists indetermining quantitative tools for managing unvertainty (in a financial insurance environment).

I. FINANCIAL, ACTUARIAL AND ECONOMIC ASPECTS OF MANAGING UNCERTAINTY

A. *Risk measures, valuation principles, and Value-at-Risk*

We will introduce a distinction between risk measures and valuation principles such as premium principle, allocation principle, solvency or capital principle, where the capital might be a regulatory, a management or rating capital. The difference between a risk measure and a principle stems from the different levels on which they operate. Indeed a risk measure is a functional that assigns a real number to a random variable (a risk) based on a set of axioms S. A principle is a `derived' functional that assigns a real number to a random variable. This separation based on an economic paradigm, is so far neglected in the framework of risk measures that appear in finance and insurance. This new approach to the management of uncertainty in financial industry has far reaching consequences and is potentially a great source of fundamental research questions. While for mathematical tractability any appropriate set of axioms S can be chosen to characterize risk measures, they mainly serve to determine the cost of a financial decision from a management point of view. The derived quantities such as e.g. capital principles then are deduced through an optimization procedure. By using several types of risk measures simultaneously for different economic approaches (management will consider a rate as cost of risk capital, while the expert will calculate the residual risk by means of another risk measure) the derived principles might be more realistic because they include several management parameters (such as amount of initial capital, premium income, tolerance level, etc) than the mechanisms that are considered so far.

Within the actuarial-financial approach of insurance business the main research question, introduced recently, consists in the determination of financial insurance streams generated by insurance portfolios and insurance companies as a whole. For that purpose, stochastic cash flows emerge discounted with stochastic interest rate models. Until recently only numerical simulation generating empirical distributions by means of scenario testing was available and to some extent applied within a financial insurance context. The reason comes from the difficulty in deriving specific analytical or numerical schemes to derive the distribution functions of the cash flows under consideration. Considering comonotonic risks, which have an economic financial justification, can circumvent these difficulties. As a consequence, reliable tails for the distribution of cash flows are obtained which provide us with the probabilities in the framework of IAS19 accounting standards with respect to fair value and supervisory value, taking into account financial pricing of the relevant cash flows.

Value-at-Risk and risk measures are mainly suitable to manage market risk. Today, financial firms are increasingly using market-value accounting for certain business lines including trading books of listed derivatives. For business lines accounted on an accrual basis (it includes some of the most traditional insurances activities), techniques of Asset-Liability Management (ALM) are more appropriate (and will be soon required by the regulator). Asset-liability management when a Brownian motion with drift describes the surplus of the company has received considerable attention. When the cash flows are unbounded, the Hamilton-Jacobi-Bellman (HJB) dynamic programming principle leads to a local time problem with smooth pasting conditions and the modified surplus is a reflected diffusion. In a recent paper, Gerber and Shiu (2003) propose to model the asset and liability values of a firm by means of correlated geometric Brownian motions (GBM). Using homogeneity property, they are able to reduce the optimal (unbounded) dividends strategy to a one-dimensional problem and to conjecture the optimality of their results. For finite-time horizon, Gerber and Shiu (2003) obtain a partial differential equation for which they are unable to provide a solution. However, a simple transformation reduces the partial differential equation to a spectral problem and suggests the nature of the modified processes, see Decamps, De Schepper and Goovaerts (2004). We propose to study the modified asset and liability values of the firm using the theoretical results of Ikeda (1961), Sato (1965) and Portenko (1978) on multi-dimensional diffusions subject to general (non-local) boundary conditions on hyper-surfaces. Those results will enable us to represent the cash flows involved by means of the intersectional local time and to understand the homogeneity properties in case of GBM. We propose to explore two-dimensional HJB principle in conjunction with more general oblique (and even non-local) smooth pasting conditions. We hope to prove the conjecture of Gerber and Shiu (2004) and to extend their results to more realistic processes for the assets and the liabilities. We think here especially about Lévy process driven models (see Schoutens (2003)), where a partial integro-differential equation (PIDE) could be expected as the central equation to be solved. Recently, in the context of American option pricing problems, numerical techniques for solving such PIDEs were developed. Non-local boundary conditions will allow for asset-liability strategies for which the asset and liability values are started afresh on another point of the boundary whenever the assets are about to fall below the liability value of the firm. Numerical solution of the two-dimensional HJB dynamic program is also an interesting issue we want to address. For what concerns financial applications, it is our hope that the present research proposal will provide a flexible framework to price hybrid and convertible securities. Indeed, the recent results on perpetual convertible bonds presented by Rogers at the Bachelier Conference exhibit many similarities with those of Gerber and Shiu (2003). We expect the conversion premium to be related to the intersectional local time of the bond price in the vicinity of the share value, just as the early exercise premium for American style derivative is related to the local time on a curve (the early exercise frontier), as shown by Peskir (2002).

B. *Measuring solvency risk*

Driven by ongoing evolutions concerning changes within the financial services market and driven by the rapid development of international insurance accounting standards that move in the direction of adopting a uniform accounting approach, see IASB (1999), there is an urgent need for research concerning the development of consistent solvency frameworks for the insurance industry and supervision as well as its economic consequences. The current approach to solvency lacks a sound scientific framework. This traditional approach uses market averages and sets solvency capital as a percentage of premiums or claims experience. On the other hand, recent research on solvency matters mostly focuses on an overall axiomatic approach. The main criticism on the overall approach is that is does not enough take into account the particular situation at hand; see e.g. Dhaene, Goovaerts and Kaas (2003), amongst others.

The current EU-regulation, i.e. the Solvency I Directive, focuses only on the pure financial position of an insurance company. It does not take into account any qualitative risk. The already launched Solvency II Project aims to go beyond the present solvency supervision scheme combining both the quantitative and qualitative risk aspects. However, the Solvency II Project is for the moment only in a very early stage. In the near future, common accounting standards will be introduced across the EU in the form of the Insurance Balance Sheet Directive. To achieve the goal that European insurers operate more transparently it is required that the new unified solvency supervision model is in accordance with these standards. It is necessary to enforce research on the `novel’ approach to the overall financial situation of insurance enterprises' analyzed from a multi-parties point of view. Developing consistent solvency frameworks and setting up evaluation procedures is a multidisciplinary task. It requires a joint approach of the following research fields: Actuarial science (quantitative and analytical aspects of solvency); Law (Juridical environment at both EC- and country-per-country level); Accounting (international accounting standards); and Economics (with a special emphasis on finance).

An academic approach, combining the different disciplines, is essential to come to a convergence of the different viewpoints.

The development of a consistent solvency framework starts by identifying the types of risk to which insurers are subjected.

The inherent variety and complexity of insurer risks suggest that the application of uniform formulae for solvency assessment across insurers can not adequately reflect their individual situations. The development of individual internal models therefore is important for an appropriate solvency assessment of insurers.

The currently ongoing discussions on banking and insurance solvency supervision all adapt a so-called `three pillars approach'; see IAA (2004). The first pillar constitutes the setting of minimal capital requirements for each company. These requirements are based on the risk exposure of the company at hand. Each company is allowed to measure its risk exposure by its own internal model. The second pillar consists of a supervisory review of each institution's risk-assessment procedures. The third pillar consists of a greater market disclosure of each institution's financial condition so that market discipline can become a powerful force compelling excessively risky firms to lower their risk exposure.

Any protection scheme has an `economical cost', to be paid by the parties involved. Increasing the safety level has a cost. The question is `what is an acceptable cost level' and how to divide these costs among the different parties involved? Optimality criteria that balance the two conflicting criteria of increasing capital (in order to decrease the probability that insolvency occurs) and decreasing capital (in order to decrease the cost of capital), have to be developed.

C. *Managing financial and actuarial risk in unit-linked life insurance*

In classical life and universal life insurance, as considered in Bowers et al (1997), the insurance company guarantees an interest rate and hence, takes the investment risk. The company invests the benefit reserves and decides on the allocation of these reserves to different types of assets (within certain restrictions imposed by the regulator). However, the public has become more aware of investment opportunities outside the insurance sector, particularly in mutual fund type investments. Unit linked life insurance (also called variable life) has been developed to offer the policyholder investment opportunities in conjunction with mortality protection. The policyholder decides the way his reserves are invested. His premiums, less expenses and mortality charges, are used to purchase units of an investment fund, which is some combination of stocks, bonds, money market funds and eventually other investment instruments. The cash value of the account at any time is determined by the number of units and the value of the purchased units at that time. The downside of this investment potential is of course that the investment returns may turn out to be very low or even negative, which will result in a lower value of the account. Therefore, the death cover of a unit linked insurance contract typically contains guarantees such as the death benefit equals the maximum of the cash value of the account and the premiums that have been paid already. Also the payment upon survival at contract termination usually contains a guarantee in terms of paid premiums. Flexible unit linked insurance (also called variable universal life) combines the flexibility of universal life and the investment potential of unit linked. Hardy (2003) considers the following issues that are important for actuaries involved in the risk management of unit-linked insurance:

1. What price should the policyholder be charged for the benefit guarantee?
2. How much capital should the insurer hold in respect of the benefits through the term of the contract?
3. How should this capital be invested?

These three issues are crucially interrelated and are topics of ongoing research. Concerning the pricing of the (death and survival) benefit guarantees, several approaches are possible. Within the actuarial approach, the distribution of the guarantee payment is converted into a quantile-based price. Other possible approaches are reinsurance (where the insurer buys the options involved) and dynamic hedging (the replicating portfolio approach). In the current actuarial/financial literature, the actuarial approach of determining premiums for the embedded options in a unit-linked insurance product is solved within a simulation framework. Recently, `comonotonic' approximations have been developed for solving reserving and pricing problems related with random cash flow streams, see e.g. Dhaene, Denuit, Goovaerts, Kaas and Vyncke ((2002a), (2000b)), as well as for solving portfolio selection problems, see Dhaene, Vanduffel, Goovaerts, Kaas and Vyncke (2004). This comonotonic approach can be adapted in such a way that it can be used for solving the issues 1, 2 and 3 mentioned above. The advantage of the comonotonic approach is that it leads to accurate, easy to compute analytical solutions, avoiding time-consuming simulation. In the existing literature on unit-linked insurance often simplifying assumptions are made (such as the insured survives with probability one until expiration). These assumptions allow for easier calculations, but transform the unit-linked insurance product to a pure financial product, and hence, are inappropriate from the insurance point of view. We believe that the theory on comonotonicity will be able to give (approximate but accurate) answers to the main issues involved in the risk management of unit-linked insurance.

Also of importance in this context is the choice of the stochastic model for the behavior of the involved financial assets. Typically, not really realistic models (Black-Scholes) are nowadays used, which underestimate the probability of extreme events. Precisely these events will largely influence the pricing and solvency (see Campolongo, Cariboni and Schoutens (2004)). In the financial world, more realistic models incorporating jumps (extremal events) and stochastic volatility are finding their way into the business (see Schoutens (2003)), and these models should be implement in the hybrid insurance-finance models. Moreover, the comonotonic approach has proven to work very well for the hedging and pricing of exotic derivatives under the more advanced models in a financial setting (see Albrecher, Dhaene, Goovaerts and Schoutens (2004) and Albrecher and Schoutens (2005)).

D. *Market Microstructure and Empirical Banking*

In the market microstructure literature, liquidity is generally viewed as a positive characteristic both for traders and markets. A liquid market is a market in which buyers and sellers can trade into and out of positions quickly without large price effects (see e.g. O'Hara (2003) or Pastor and Stambaugh (2003)). Typically, four dimensions of liquidity are considered: width, depth, immediacy and resiliency. The first three dimensions received considerable interest (see e.g. Degryse (1999) for an empirical application). Resiliency, in contrast, has not yet obtained much attention. In a recent paper, Foucault et al. (2003) define it as the speed of recovery of the market (in terms of prices, depths and spreads) after a relatively large shock. Following an order classification procedure introduced by Biais, Hillion and Spatt (1995), recent empirical work by Degryse et al. (2002) tackles resiliency by looking at order flow characteristics following aggressive orders (orders that move prices). All papers, however, study resiliency assuming idiosyncratic shocks. Hasbrouck and Seppi (2001), however, point to common factors in other dimensions of liquidity. An interesting extension of the previous papers is therefore to look at commonality in resiliency.

Another promising venue in the market microstructure literature is to focus on liquidity measures in stress periods versus normal procedures, where stress periods are defined as periods with high volatility. Whereas theory until recently focused on liquidity provision within one trading system, more recent work aims to focus on liquidity provision across differently organized trading systems. This again raises a few interesting and important questions. How do these ``combined" mechanisms perform vis-à-vis the basic trading systems? How can key players on certain markets manipulate the other market? The availability of high-frequency data for different trading systems allows to address whether particular trading systems are more resistant to stress periods, or whether stress periods are transmitted from one trading system to another.

Empirical banking also deals with information problems. A raison d'être for financial intermediaries is resolving asymmetric information. Recent empirical and theoretical work deals with issues of how banks and their internal organisation influences the use of ``soft" and ``hard" information (Degryse and Ongena ((2004a), (2004b)) and Stein (2002)). An interesting unresolved issue is how information gathering within and across banks influences the statistical properties of loan pricing. Theoretical work by von Thadden could be an interesting starting point.

E. *Enterprise Risk Management (ERM)*

Enterprises operate in environments where factors such as globalization, technology, regulation, restructurings, changing markets, and competition create uncertainty. Uncertainty originates from the inability to precisely determine the likelihood that potential events will occur and the associated outcomes of these events. Managements of some companies (and other entities) have developed processes to identify and manage risks across the enterprise. While considerable qualitative information on enterprise risk management is available, guidelines in developing effective quantitative risk management architecture are hardly available. The Committee of Sponsoring Organizations of the Treadway Commission (COSO) issued a conceptual framework providing integrated (qualitative) principles on Enterprise Risk Management (ERM/COSO 2003). Also, in the aftermath of the Enron-Andersen scandal, the Sarbanes-Oxley Act (2002) in the US includes a section (the famous Section 404) that requires, amongst other things, management of listed US firms to assess the quality of their control and risk management processes and to provide a written statement about this. This requirement also has implications for all subsidiaries of US listed firms worldwide.

The qualitative guidelines about enterprise risk management in the COSO/ERM framework are widely accepted throughout business community. They are also adopted by the major public accounting firms and are referred to in risk management advisory services and internal control consultancy. Although risk management is hype in the business world nowadays, scientifically backed methodological guidance in this matter is extremely scarce.

According to COSO/ERM, an enterprise's risk responses fall within the following categories (see also Knechel (2002)):

- Risk avoidance - Action is taken to exit the activities giving rise to risk (e.g. risk avoidance may involve exiting a product line, declining expansion to a new geographical market, or selling a division.
- Risk reduction - Action is taken to reduce the risk likelihood or impact, or both. This may involve any of a myriad of everyday business decisions.
- Risk sharing - Action is taken to reduce risk likelihood or impact by transferring or otherwise sharing a portion of the risk. Common risk-sharing techniques include purchasing insurance products, pooling risks, engaging in hedging transactions, or outsourcing an activity ·
- Risk acceptance - No action is taken to affect likelihood or impact.

An academic approach to the ERM paradigm is essential to further enhance corporations' risk management attitudes, and requires a combination of different disciplines including auditing and control, finance, actuarial science, economics and mathematics.

To assess an enterprise's risk theoretically sound probabilistic models are needed. Examples of probabilistic models include value at risk, cash flow at risk, earnings at risk and the development of credit and operational loss distributions. See also Section I.A. Development of loss distributions related to both financial and non-financial hazards (events) is needed.

Management also needs to assess how events correlate, where sequences of events combine and interact to create significantly different probabilities or impacts. While the impact of a single event might be small, a sequence of events might have more significant impact. Looking at interrelationships of risk likelihood and impact is an important management responsibility. Effective enterprise risk management requires theoretically sound tools to achieve this.

II. STATISTICAL CONCEPTS FOR MANAGING AND MODELING UNCERTAINTY

A. *Structural breaks*

Structural breaks such as jumps can occur in the trend of a series, but also in the volatility. Breaks can appear only in the trend, or only in the volatility, but they can also occur simultaneously in both (since mean and variance functions are closely related). So far, modeling of structural breaks has been done mainly by relying on appropriate parametric models, and often under the assumptions that breaks do not occur simultaneously in the mean and the variance function. However, it is not always easy to know what are appropriate parametric models, and the appearance of breaks in both the mean and variance function is a realistic situation that has been left quite unexplored so far. Nonparametric and semi-parametric methods are needed when appropriate parametric modeling is not available yet. Nonparametric methods for detecting change points in the mean (regression) function for independent data are reasonably well studied by now. The performance of such methods depends crucially on the choice of smoothing parameters. Flexible methods should be such that they do not need to know the number of change points in advance. See for example Gijbels and Goderniaux (2004). Methods for detecting change points in univariate (mean) functions can, with extra efforts, be extended to detecting change points in a bivariate (or multivariate) function. See for example Gijbels, Lambert and Qiu (2004). Most papers dealing with detection of change points in the mean function assume implicitly that the variance function is continuous. However, when breaks occur both in the mean and the variance function then this possibility should be modeled appropriately, in order to make their detection more powerful. See for example Gao, Gijbels and Van Bellegem (2004). It is clear that a good knowledge of smoothing methods and their applications is necessary.

The occurrence of structural breaks can be blurred by the appearance of outlying or aberrant observations. Hence there might be a need for robust methods to detect structural breaks. This issue did not get any attention so far.

In financial applications when, for example, modeling the uncertainty in the stock price behavior, the inclusion of jumps in a model is often crucial in explaining the related option prices observed in the market. See Campolongo, Cariboni and Schoutens (2004).

B. *Estimating frontiers and boundaries*

For the purpose of estimation of the productivity of companies, modeling via by frontier curves, representing the maximum attainable output for a given input, has been proven very efficient. The data come into the form of a multivariate cloud of points, each point representing the productivity of one company. This problem can be translated into the problem of estimating the boundary curve of the support of a multivariate density function. One may wish to address such a frontier estimation problem with flexible nonparametric methods that do not rely on the unrealistic assumption that all companies produce under the theoretical optimal efficiency curve (which is implied by nonparametric frontier estimation methods that are restricted to the case of deterministic frontiers). Possible approaches here are also to use extreme quantiles or order statistics. See for example Gijbels, Mammen, Park and Simar (1999) and Gijbels and Peng (2000).

The problem of estimating boundary points is also closely related to that of detection of discontinuity points of a curve (read break points). A boundary point is in fact a special case of a discontinuity point. The techniques for both problems, change point detection and boundary estimation, show often similarities, and these should be more exploited.

Finally, the estimation of a stochastic frontier function is in fact a deconvolution problem, since due to the stochastic nature some observations might fall above the theoretical boundary. Dealing in an adequate way with such a deconvolution problem is not an easy task, as can be seen from for example Delaigle and Gijbels ((2002), (2004a), (2004b)). When one needs to deal with measurement errors when estimating boundaries this becomes even a harder problem. Many practical and theoretical issues still need to be addressed there.

C. *Events, extreme events, and their prediction*

The occurrence of extreme events is important in actuarial and financial business. In actuarial applications one often thinks in terms of large claims and losses. One aspect in modeling of large claims is looking at the excess of a claim above a certain (high) threshold. An appropriate way of modeling extreme events is via heavy-tailed distributions. When analyzing large claims/losses data, important characteristics are extreme quantiles and the tail index (which measures the heaviness of the tail of the distribution).

In finance, extreme value theory is of importance in the modeling of crashes, risk-management issues and default/credit risk. In Cariboni and Schoutens (2004) Credit Risk models (driven by jumps) are proposed, which have the ability to reconstruct observed market prices of Credit Default Swaps.

Loss reserving, for example, is a prediction process: given the data, we try to predict future claims. The prediction error is a measure of precision of these estimates. This can be useful to set up safe reserves. Although the computation of the prediction error is a good starting point, it is only a measure of the second moment of the predictive distribution. Other measures, such as skewness, risk measures and extreme percentiles of the total distribution, are also very important. The final objective of the analyst is to understand the whole predictive distribution of the reserves and derive the right statistical measures of it.

A typical claims reserving exercise starts with a statistical analysis of the historical claims in the dataset in order to build an appropriate model. An important step in the development of stochastic reserving techniques was the introduction of a loglinear model with parameters that allow to model trends in a run-off triangle in three directions (horizontally, vertically and diagonally). Distributions used to describe the claim size should have a subexponential right tail. Furthermore, the phenomena to be modeled are rarely additive in the collateral data. A multiplicative model is much more plausible. Working with ordinary linear models cannot solve these problems; one needs generalized linear models.

Via bootstrap techniques one can create a huge number of pseudo-databases, consistent with the same underlying distribution, to gain insight in the predictive distribution via a two-stage simulation. An alternative to bootstrapping is simulation from the joint distribution of the model parameters. In this way one can obtain, again via a two-stage simulation technique, a predictive distribution for every cell in the unobserved part of the run-off triangle. For generalized linear models with a non-normal error structure and a non-identical link function, this technique is not straightforward. The use of Bayesian techniques can be a possible solution, making assumptions about the prior distribution of the parameters and finding their joint distribution, given the data. Via simulation and numerical techniques Markov Chain Monte Carlo techniques can deliver a way out, when this posterior distribution cannot be expressed analytically. Another interesting approach is the use of the comonotonicity concept in order to derive approximations for the distribution function that are larger or smaller in convex order sense than the exact distribution.

A few relevant references in this area are: De Vylder and Goovaerts (1979), Goovaerts and Redant (1999), Antonio et al. (2004) and Hoedemakers et al. ((2003), (2004a), (2004b)).

Traditional loss reserving techniques in non-life insurance are based on so-called run-off triangles that contain aggregated figures (per arrival and development year combination). As mentioned above, both observed data and future observations usually are modeled in a parametric way using a general or generalized linear model. However, such a run-off triangle provides only a summary of an underlying database consisting of individual claim figures and related covariables. Therefore Antonio et al. (2004) suggest modeling the individual data and they interpret the available data in the framework of longitudinal data. Making use of the theory of linear mixed models, a flexible loss reserving is built which allows modeling and prediction of individual payment profiles.

Apart from the issue of loss reserving, other typical problems from actuarial statistics (e.g. credibility theory) lead to longitudinal data. Therefore we plan to use recent techniques from longitudinal data analysis to model actuarial data. As such we work on further integration of traditional actuarial techniques in a broad statistical context. For instance, generalized linear mixed models will be considered in the context of individual loss reserving and as a framework to interpret traditional credibility estimators. See Antonio and Beirlant (2004). Another typical problem is the fact that longitudinal actuarial databases very often contain an abundant number of zero observations (i.e. no claim occurred). General and generalized linear mixed models can then be used to model explicitly the occurrence of such zeros. All our models will be developed both in a likelihood-based and Bayesian way.

Obviously, the use of adequate families of heavy-tailed distributions gets a lot of attention in this field, as is the statistical inference under these families. See for example Mathijs, Delafosse, Guillou and Beirlant (2004) and Beirlant, Goegebeur, Segers and Teugels (2004).

First insights can often be gained from simply looking at data and summarizing them via descriptive statistics such as box plots. When dealing with skewed distributions, or distributions for which a lot is happening in the tails, the classical box plots provide too few details about what happens in the important tails. Hence adjusted box plots for such distributions are a helpful graphical tool, as has been demonstrated in Vandervieren and Hubert (2004). So far, this adjusted box plot incorporates a measure of skewness (Brys et al. 2004a). For extreme value distributions the display could be even improved by adding robust measures of tail weight (Brys et al. 2004b) or a robust estimator of the tail index, as introduced in Vandewalle et al. (2004).

Modeling the occurrence of extreme events can be improved by including information from covariates or independent variables. This has been investigated in Beirlant and Goegebeur (2003). To obtain more robust results, we can extend the integrated squared error approach as in Vandewalle et al. (2004) to the regression setting. In the next stage, we want to consider the estimation of the extreme value index and extreme quantiles in the presence of censoring, where observations are measured within a restricted range of values. This type of data often occurs in insurance when reported payments cannot be larger than the maximum payment value of the contract. Robust alternatives of the estimators developed in Beirlant et al. (2004) can be constructed by combining the methodology of Vandewalle et al. (2004) with the robust quantile estimators described in Debruyne and Hubert (2004).

D. *Modeling dependencies between variables*

The uncertainty in a process is influenced by several variables and these variables can be related to each other in various ways, and in various degrees of strength. Modeling the dependence structure between two or more variables is crucial in investigating risk and managing it. Important tools for modeling dependence are copulas. A copula explains how the joint distribution of the variables is related to the marginal distributions of each of the variables. Apart from trying to model the dependence structure between variables via appropriate families of copulas, one can also study certain properties that describe dependence structures between variables. We just name a few: comonotonicity, countermonotonicity, convex ordering, stochastic ordering, etc. These concepts are particularly useful in modern actuarial risk theory when evaluating the appropriateness of risk measures. See also Sections I.A and I.C. Some interesting statistical research topics in this area are: non-and semiparametric estimation of copulas, under possible qualitative restrictions, estimation of copulas based on censored observations (see also Section II.C). In finance, copulas appear to be very important in a credit-risk setting i.a. for the modeling of the dependence in Collateralized Debt Obligations (CDO's). A few relevant references in this area are: Albrecher, Dhaene, Goovaerts and Schoutens (2003), Denuit and Dhaene (2003) and Goovaerts, Kaas, Laeven and Tang (2004).

Dependencies can of course also occur on a more elaborate level, such as dependencies between time series. Aspects and research issues of statistical modeling of such causal relations in time are discussed in Section II.F.

E. *Estimation and testing of models with non-tractable likelihood*

There are many examples where simple parametric modeling assumptions lead to a non-tractable likelihood function. These include, among others, disequilibrium models, commodity storage models, and models with dynamic latent variables such as the stochastic volatility (SV) model popular in finance. Likelihood-based analysis in the latter class of models is precluded by the presence of a high-dimensional integral in the likelihood function. This seriously complicates the analysis, regarding both estimation and testing of such models. One has to resort to other methods like GMM, simulated ML, MCMC (Chib et al. (2002)), or indirect inference (Gouriéroux et al. (1992); Dhaene et al. (1998)). Most of these methods are simulation-based and do not yield closed-form solutions for the problems of estimation and inference. Exceptions are the indirect inference methods of Dhaene (2004) and GMM-based methods of Dhaene and Vergote (2004). We intend to develop these analytical results further in the following directions:

(i) relaxing the parametric assumptions, which will lead to semi-parametric inference for SV models;

(ii) incorporating dependence between the mean equation and the volatility equation to capture the so-called ``leverage effect" often observed in financial data;

(iii) extending the methods to estimate continuous-time SV models with observations that are non-equidistant in time. The latter extension would make it possible to estimate SV models using transaction data, a hot topic in the current financial econometrics literature.

F. *Multivariate modeling of causal relations in time*

In many financial applications, the object of interest is a time series. When managing uncertainty, it is of interest to know whether certain time series contain valuable information on the future values of other series of interest. If one series is ``causing" the other, it can be used to obtain better forecasts, hereby reducing uncertainty. To assess the causality between two processes, one usually refers to the well-known concept of Granger causality, introduced by the 2003 Nobel Prize winner in Economics. Granger causality reflects the extent to which a process is leading another process, and builds upon the notion of incremental predictability. Investigating causality is a topic of main interest in scientific research. In the financial literature, Granger Causality analysis has been applied, for instance, to identify price-leadership patterns among national stock prices, to study the stock price-volume relationship, to get insight in the dynamic behavior of bonds and stocks, or in the international links between interest rates.

The concept of causality is standard in the case where both series are univariate time series. In many applications, however, detecting and quantifying causal relations between multivariate time series is highly pertinent. For example, one might be interested in knowing whether the trading volumes of a collection of stocks are Granger causing future returns of this collection of stocks. Cross-series relationships need to be modeled and taken into account. An example is Lemmens et al (2004), where the predictive content of European business surveys for actual production accounts is studied. In a multivariate setting, Causality is studied by means of Vector Autoregressive (VAR) models (e.g. Gourieroux and Jasiak (2001)). There are, however, several limitations associated with this approach.

(i) The dimension of the series may be large, which will lead to an explosion of the number of parameters in the VAR-model. In this case, a factor model gives a more parsimonious modeling of the series. As such, Stock and Watson (2002) used this approach to predict a univariate time series using a large number of predicting variables. Aim is to study causality within a dynamic factor model framework.

(ii) We do not only want to test whether causality is present or not, we would also like to analyze the structure of this causality in more depth. As such we can study the cross-causalities: how large is the impact of a particular component of the leading series on each of the components of the series to predict. It is also of interest to study the causality structure over time: is there an impact in the very short run or only on the middle and long run? Graphical displays, presenting the results of a possibly complicated analysis in an understandable way, need to be developed for the use of the decision makers (as was done in Croux et al (2001) for studying associations between components of a high dimensional time series).

(iii) Since we are working with multivariate data, there is a higher risk for model misspecification and a larger probability that outliers might be present. One could expect presence of both outliers in the time dimension as in the cross-sectional dimension. To make the causality analysis robust to outliers, we propose to use a robustly estimated factor model, as in Croux et al (2003) and Hubert et al (2004), but now in the dynamic setting. Good diagnostics for outlier detection need to be developed. Another major problem is the selection of the appropriate factor model. This could for example be done by means of cross-validation for which fast algorithms need to be developed (see e.g. Hubert and Engelen (2004)).

Within the network of excellence, the scientific expertise is present to tackle the above problems. Aim would be to develop a toolbox for detecting and measuring causal relations between collections of large numbers of time series.

G. *Multivariate semi-affine models of the stochastic discount factor*

The pricing of financial claims and liabilities crucially hinges on the specification of the stochastic discount factor (SDF). This general specification problem consists of two parts: the identification of the interest rate dynamics and the selection of the types of (priced) risks in the economy. Conditional on the specification of the interest rate dynamics, the types of risk (under suitable regularity conditions), the historical and risk-neutral measures are well defined, allowing for the valuation of any type of financial asset (e.g. Sections I.A – I.C).

Within the finance literature, the class of multivariate affine models has been used extensively as a specification of the SDF (Duffie and Kan (1996) and Dai and Singleton (2000)). This class of models results in a linear dynamic latent factor model, allows for interdependence between the factors and is in general sufficiently flexible to price financial assets.

The main drawback of this approach is that the models in the affine class do not incorporate economic equilibrium conditions. Both the interest rate dynamics and the sources of risk are ultimately economic concepts and could be modeled as such. Recently, a class of models based on economic equilibrium conditions (Euler equations) of standard macroeconomic models has been developed addressing this issue. Log-linearization of the Euler equations in conjunction with QZ decompositions result in structural, i.e. based on economic equilibrium, affine models for the discount factors both in a discrete and a continuous-time framework (Dewachter and Lyrio ((2004a), (2004b)) and Dewachter et al. (2004)). These models allow for both latent and observable macroeconomic factors and retain the linearity of the affine class. One objective of this project is to extend the class of structural models to the semi-affine class, allowing for nonlinear interdependencies between the macroeconomic factors. Linear-quadratic (or higher order) expansions of the Euler equation identify the structural macroeconomic interrelations within the SDF. Subsequently, we will develop feasible (non-linear) filtering and/or method of moments procedures (GMM, SMM) to estimate the SDF dynamics. The results and procedures from other parts of this project (e.g. Sections II.D --- II.F) will be useful in the estimation and/or the evaluation of the performance of this class of models.

Finally, we intend to apply this type of model to the valuation of demand deposits. The financial stability of the banking system hinges on an appropriate valuation of this type of asset. As demand deposits fluctuate primarily with macroeconomic developments (see Jarrow and Deventer (1998)) the type of models developed above are appropriate. Given a proper valuation scheme, optimal investment strategies, duration matching and replicating portfolios can be obtained.

H. *Minimum risk model selection, averaging and testing*

One of the most central decisions in modeling uncertainty is which information to include in the model. The choice of the model is crucial for all further decisions made and inferences deduced. An illustration of a set of (stock derivative pricing) models all nicely calibrated on the same data set (vanilla options), which let to complete different outcomes (exotic option prices) can be found in Schoutens, Tistaert and Simons (2004). The selection of the wrong model can have very pernicious effects and could lead to the disastrous underestimation of the risk (see Schoutens, Tistaert and Simons (2005))

Claeskens and Hjort (2003) developed a focused information criterion to guide the model choice. An essential novel contribution of this work is that the model selection is guided by a minimization of a risk function. The best model is that one where the estimated risk is minimal. Since the risk depends on the quantity of interest, focused model selection allows to accurately select a model for a particular goal. So far, these methods have been applied to fully specified likelihood models, including generalized linear models. In life insurance for example, specific interest is in prediction of survival times, preferably taking background demographic or medical information into account. We plan to develop focused information criteria specific for duration data, possibly censored. The applications are far-reaching, including model choice for the purpose of predicting survival times (for people, or also for example for stocks, …), estimation of relative risk quantities, quantiles of the survival distribution, etc. Focused information criteria are mainly based on mean squared error, for classification purposes our goal is to extend this scope and look at different measures of risk such as classification error for example. Also the problem of a large number of variables relative to the sample size needs to be addressed.

Once a model is selected, the statistical task is to further deal with the uncertainty that leads to the model choice. Hjort and Claeskens (2003) dealt with this uncertainly by studying frequentist model average estimators, which are weighted estimators, where the random weights are determined by the model selection method. The result shows that the resulting distribution of estimators-after-model selection is no longer equal to the classical limiting distribution. This has a consequence for all further inference. So far these methods are worked out in parametric likelihood models. Dealing with this issue is also crucial for example in modeling survival data where often a semiparametric Cox model is used, but also in nonparametric estimation models. The study of model uncertainty in nonparametric statistics is currently an important open problem. Methods as in Hjort and Claeskens (2003) are an essential step towards solving this problem.

If the choice of background information, variables to include, is decided upon beforehand, it is still crucial to check the validity of the model for the data at hand. Nonparametric testing methods do not require stating many parametric assumptions. Such lack of fit tests have been developed for several applications, see for example Aerts, Claeskens and Hart ((2000), (2004)), Claeskens (2004) and Claeskens and Hjort (2004). There is a definite need for a theoretical and numerical comparison of nonparametric tests. Considering the different nature of the test statistics, this comparison will be non-trivial.

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